Homotopy analysis of the magnetohydrodynamic flow and heat transfer of a second grade fluid in a porous channel

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A R T I C L E I N F O

Article history:
Received 11 March 2013
Received in revised form 22 July 2013
Accepted 25 July 2013
Available online 21 August 2013

Keywords:
Homotopy analysis method
MHD flow
Heat transfer
Second grade fluid
Porous channel

A B S T R A C T

In this paper, we analyze the MHD (magnetohydrodynamic) flow and heat transfer of a second grade non-Newtonian fluid in a channel with porous wall. HAM (Homotopy analysis method) is used to obtain analytical solutions of the governing system of nonlinear ordinary differential equations. The analytical solutions are obtained in the form of infinite series and the convergence of the series solution is discussed explicitly. The obtained results are presented through graphs for several sets of values of the parameters, and the salient features of the solutions are analyzed. A comparison of our HAM results with the available numerical results in the literature (obtained by Runge-Kutta and shooting methods) shows that our results are accurate for a wide range of values of the parameters.

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1. Introduction

A non-Newtonian fluid is one whose fluid properties differ from that of a Newtonian fluid. Most commonly, the viscosity of non-Newtonian fluids is not independent of shear rate or shear rate history. However, there are some non-Newtonian fluids with shear-independent viscosity, that nonetheless exhibit normal stress-differences or other non-Newtonian behaviors. Many salt solutions and molten polymers are non-Newtonian fluids. Also the commonly found substances such as ketchup, custard, toothpaste, starch suspensions, paint, blood, and shampoo are non-Newtonian fluids. In a Newtonian fluid, the relation between the shear stress and the shear rate is linear, passing through the origin, the constant of proportionality being the coefficient of viscosity. In a non-Newtonian fluid, the relation between the shear stress and the shear rate is not linear, and can even be time-dependent. Therefore, a constant coefficient of viscosity cannot be defined. Although the concept of viscosity is commonly used in fluid mechanics to characterize the shear properties of a fluid, it can be inadequate to describe non-Newtonian fluids. They are best studied through several other rheological properties which relate stress and strain rate tensors under many different flow conditions, such as oscillatory shear, or extensional flow which are measured using different devices or rheometers. The properties are better studied using tensor-valued constitutive equations, which are common in the field of continuum mechanics.

The studies of laminar boundary layer flow of non-Newtonian fluids have received much attention because of the power needed in stretching a sheet and the analysis of heat transfer rate in a non-Newtonian fluid are quite different from those of a Newtonian fluid. The problems on viscoelastic fluid flow have been extensively studied by various authors [1–3], which include numerous applications to several industrial manufacturing processes. One class of viscoelastic fluid is of the differential type, namely the second grade.

The first model for such a fluid is proposed by Rivlin and Ericksen [4,5] and complete discussion on this fluid model is given by Dunn and Rajagopal [6]. In Ref. [7], Kurtcebe and Erim have studied heat transfer of a second grade fluid flow in a channel with one porous wall. Very recently, a study on the laminar flow of a second grade electrically conducting viscoelastic fluid between two parallel plates has been considered by Parida et al. [8].

Magnetohydrodynamics (MHD) is the study of the interaction of electrically conducting fluids and electromagnetic forces. The field of MHD was initiated by Swedish physicist, Hannes Alfvén for which he received in 1970 the Nobel Prize [9]. In 1937, Hartmann and Lazarus [10] studied the influence of a transverse uniform magnetic field on